A closer look on uncertainty evaluations using Virtual Experiments and VCMMs

2nd International Workshop on Metrology for Virtual Measuring Instruments - VirtMet2025

3 December 2025

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Co-funded by the European Union

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Coordinate Measurement Machine (CMM)
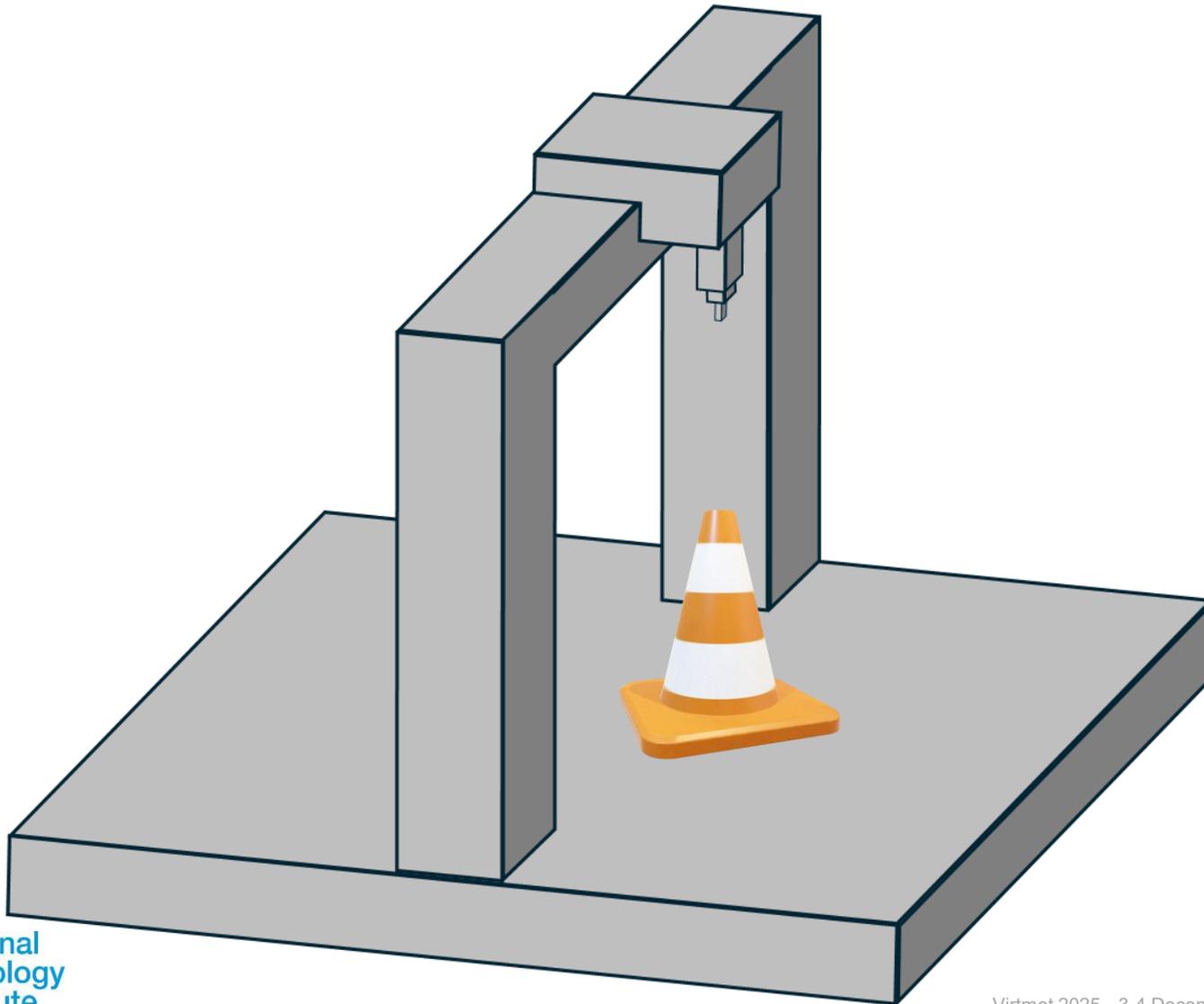
A CMM measures coordinates of an object

- Tactile or optical measurements
- Many different geometries can be measured
- Estimates of measurands determined from measured (x, y, z)-coordinates
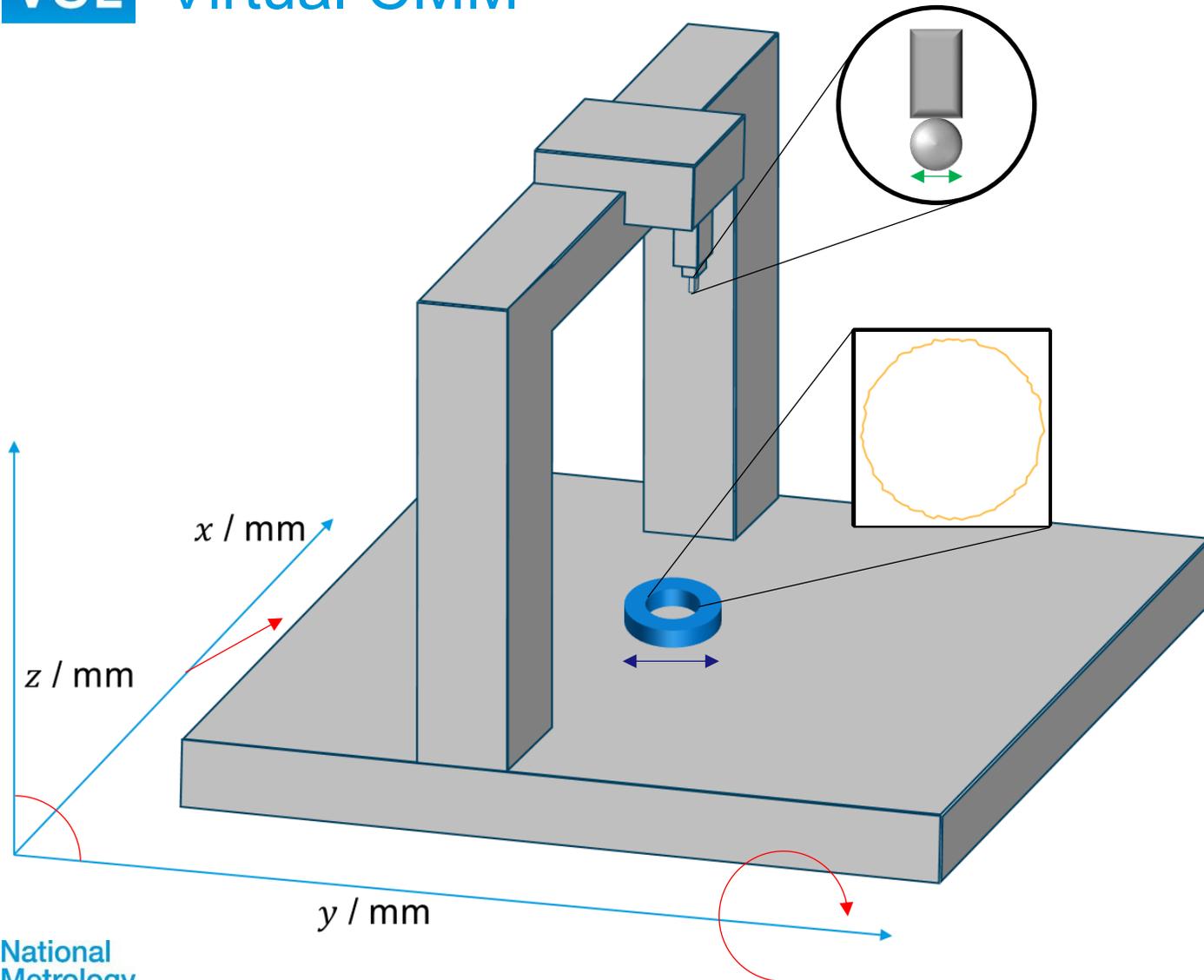
Virtual CMM

The virtual CMM models how uncertainty sources influence the measurement data

This includes:

- Kinematic errors
- Probe errors
- Workpiece temperature
- Workpiece roughness
- Random noise

What is a VCMM?

- **A detailed mathematical model of a CMM**, incorporating all relevant error sources Z, which can be used for:

  - simulating measurement data given true shape geometrical parameters Y, i.e., $\quad X_{\text{out}} = g_1(Y_{\text{in}}, Z)$

  - perturbing measurement data with additional measurement errors, i.e, $\quad X_{\text{out}} = g_2(X_{\text{in}}, Z)$

  - calculating an uncertainty associated with an $\hat{Y}_{\text{out}}$, i.e.: $\quad u(\hat{Y}_{\text{out}}) = g_3(X_{\text{in}}, Z)$

  - calculating an estimate and an uncertainty, i.e.: $\quad (\hat{Y}_{\text{out}}, u(\hat{Y}_{\text{out}})) = g_4(X_{\text{in}}, Z)$

- Different formulations are closely linked, we will not be strict in this presentation.

Typical way of calculating an uncertainty using a VCMM
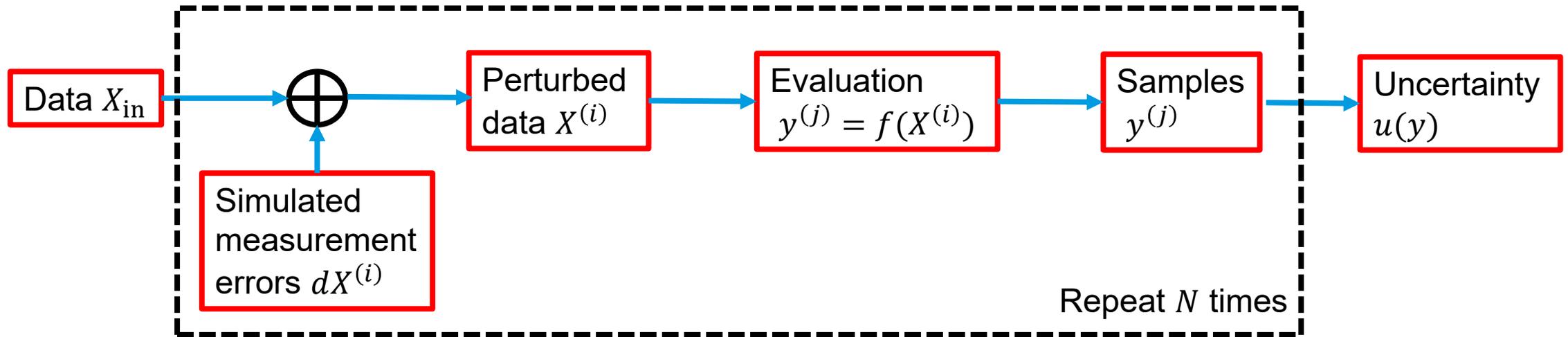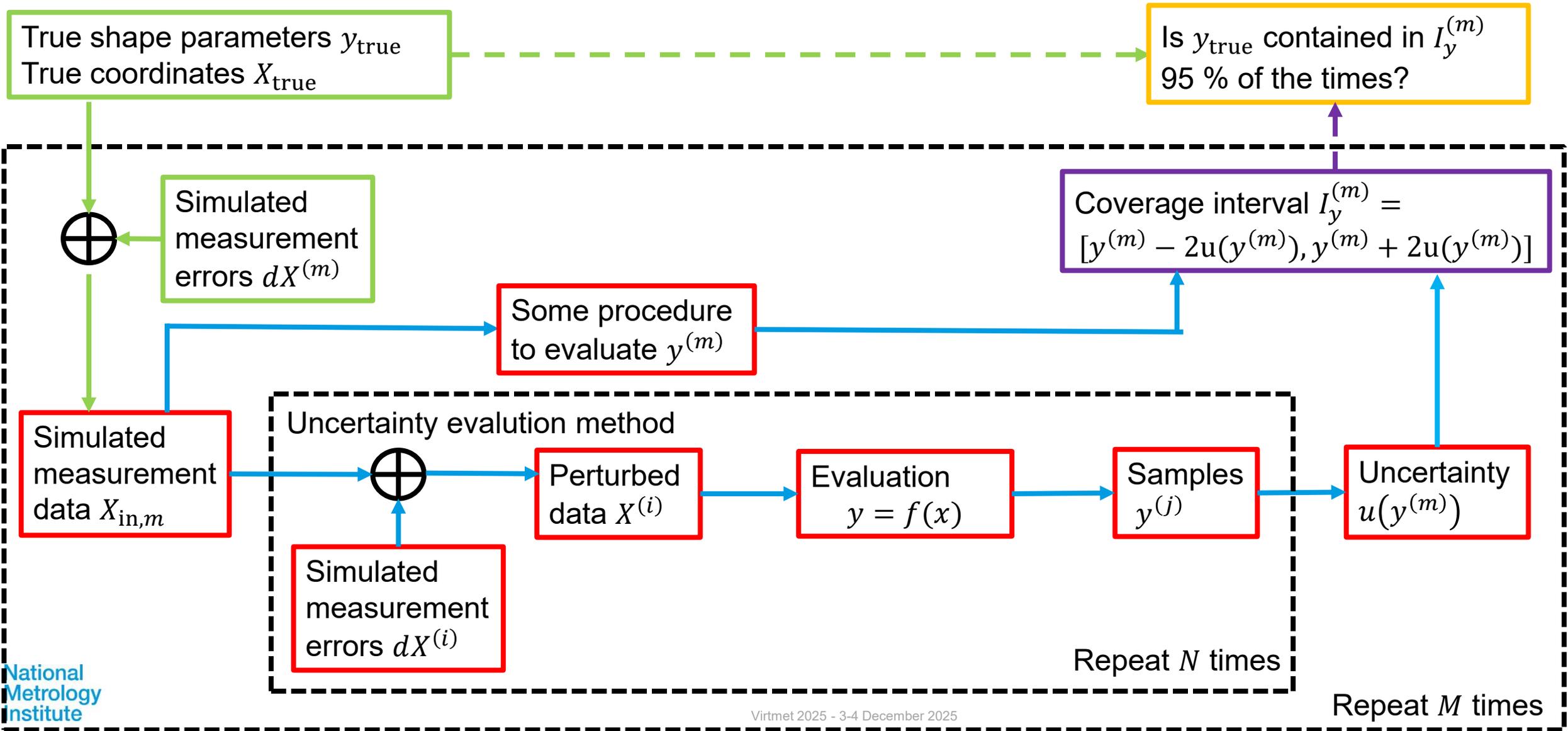

- Note: it matters what you use for input X_{in}!

  A. Measurement data

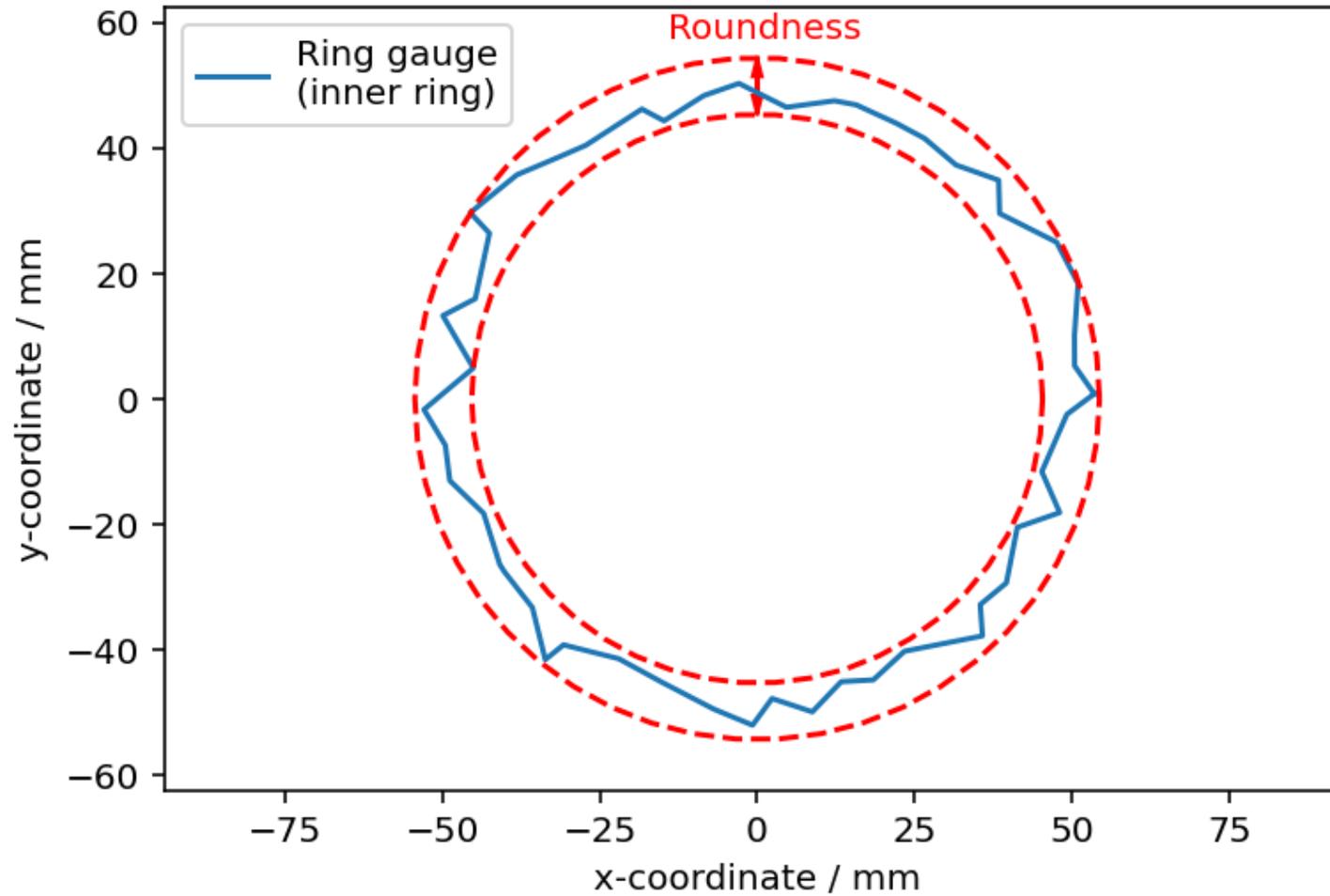 B. Nominal shape

  C. Simulated imperfect shape

See VirtMet-1 presentation &

Gertjan Kok, Gerd Wübbeler, Clemens Elster, *Impact of imperfect artefacts and the modus operandi on uncertainty quantification using virtual instruments*, Metrology, 2022, 2(2), 311-319; https://doi.org/10.3390/metrology2020019

The VCMM as statistical model & evaluation of success rates

True shape parameters y_true
True coordinates X_true

Is y_true contained in $I_y^{(m)}$ 95 % of the times?

Simulated measurement errors $dX^{(m)}$

Coverage interval $I_y^{(m)} = [y^{(m)} - 2\mathrm{u}(y^{(m)}), y^{(m)} + 2\mathrm{u}(y^{(m)})]$

Some procedure to evaluate $y^{(m)}$

Simulated measurement data $X_{\text{in},m}$

Uncertainty evalution method

Perturbed data $X^{(i)}$

Evaluation $y = f(x)$

Samples $y^{(j)}$

Uncertainty $u(y^{(m)})$

Simulated measurement errors $dX^{(i)}$

Repeat N times

Repeat M times

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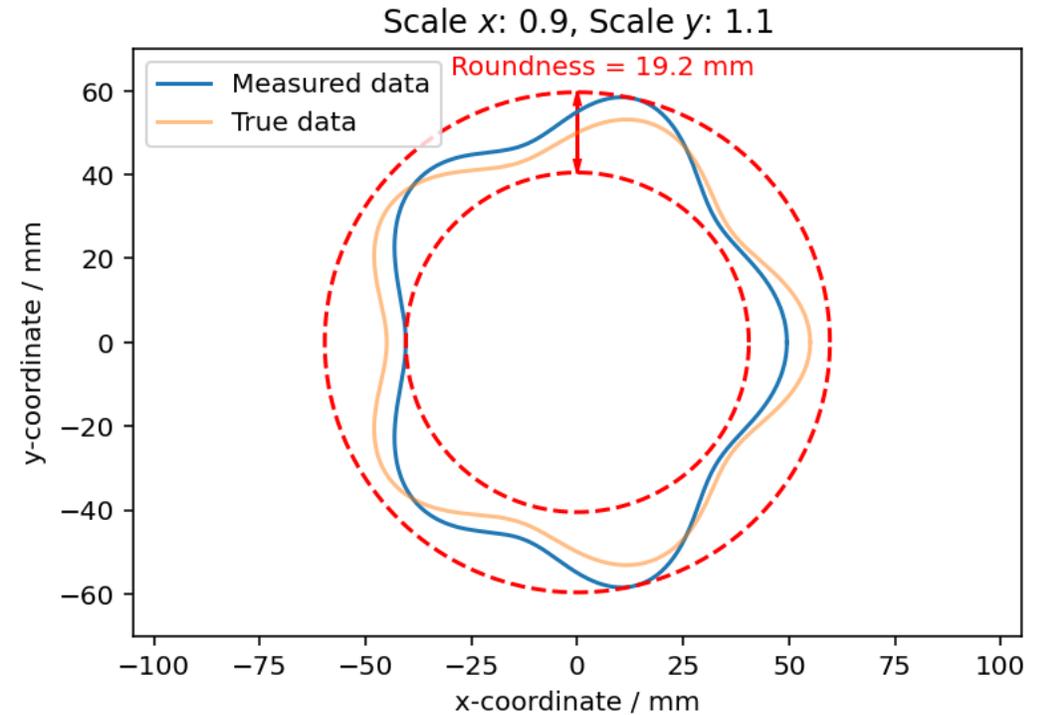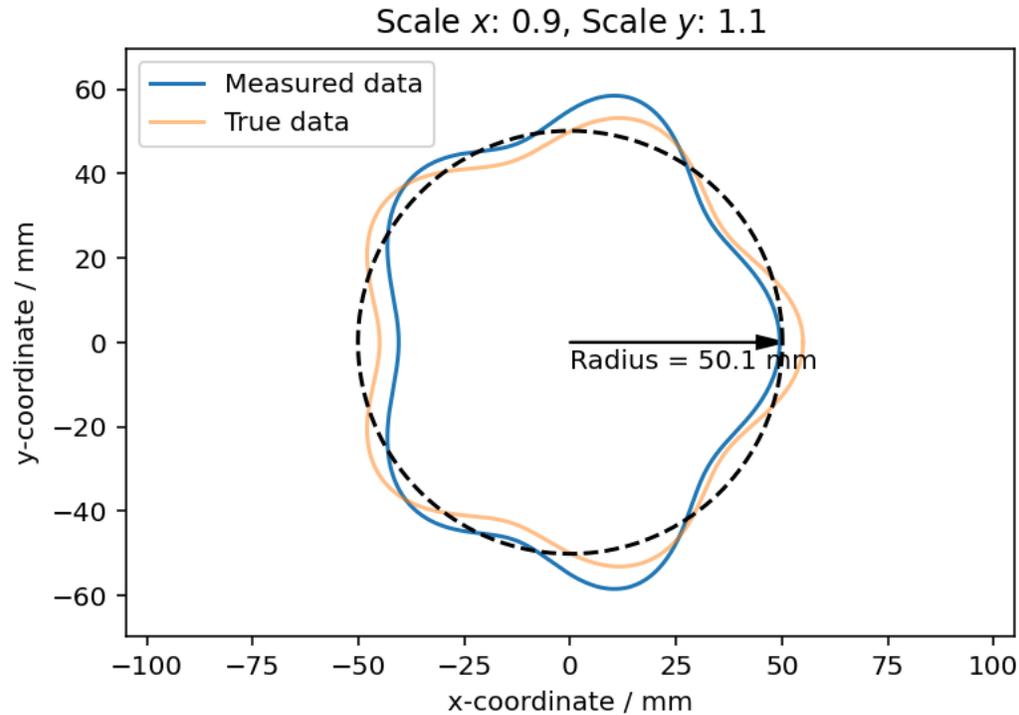
Example: ring gauge diameter & roundness evaluation

Simulation results (1)

- Coverage-% for radius is fine

- Coverage-% of standard procedure is very low for roundness

- Standard method for assessing roundness turns out to be statistically biased

Method	Coverage-% Radius	Coverage-% Roundness
Standard procedure	93 %	18 %

Bias in roundness evaluation

Radius sometimes increases, sometimes decreases due to scale errors → No bias

Roundness almost always increases when scale errors are present → Bias

Accounting for the bias: (A) correcting the estimate

correction

bias

Note: The bias should be evaluated w.r.t. the underlying ground true value in the VCMM. We assume here that it equals the estimate based on the data.

Corrected estimate

Estimate   Samples mean

Confidence interval   Confidence interval
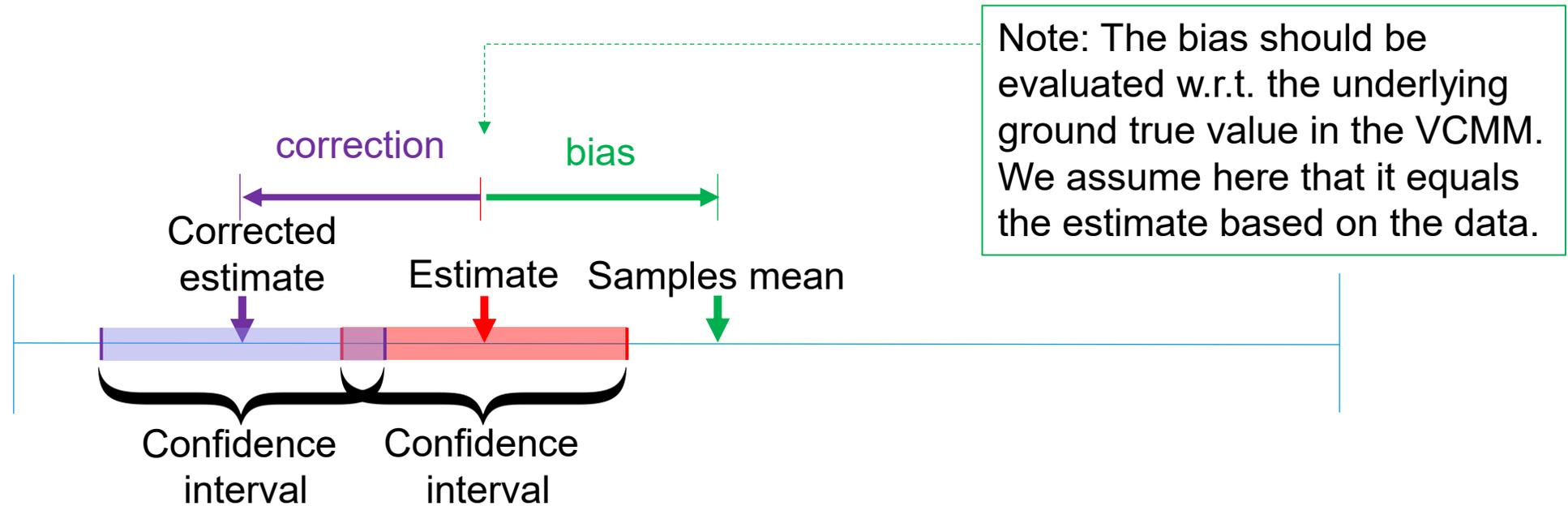
Bias can be dealt with in two ways:

- A. Correcting the estimate by subtracting bias

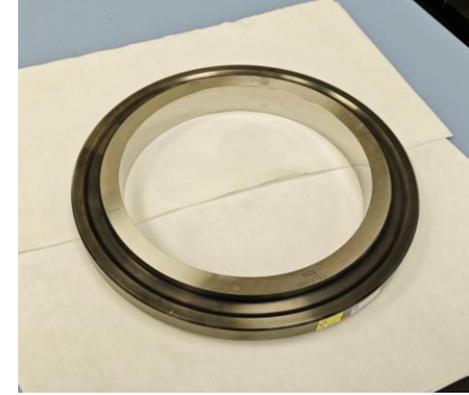# Accounting for the bias: (B) increasing the uncertainty

Lower bound equal to method with corrected estimate

Note: The bias should be evaluated w.r.t. the underlying ground true value in the VCMM. We assume here that it equals the estimate based on the data.

bias

Estimate   Samples mean

Corrected **symmetric** confidence interval
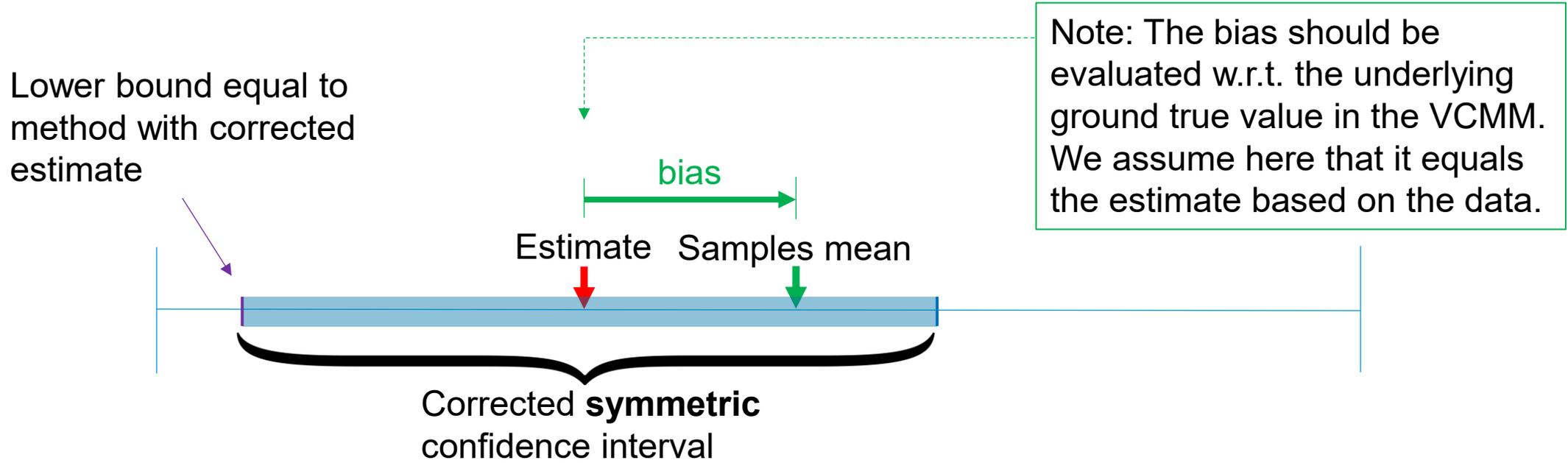
Bias can be dealt with in two ways:

- B. Increasing the uncertainty by adding the bias to the confidence interval

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Simulation results (2)

- VCMM can be used to correct the estimate for the statistical bias

- Both methods yield a coverage of approximately 95 %

- Method A has lowest uncertainty but estimate y now depends on provided uncertainties $u(z)$

- Method B has significant higher uncertainty but leaves estimate y unchanged w.r.t. standard procedure

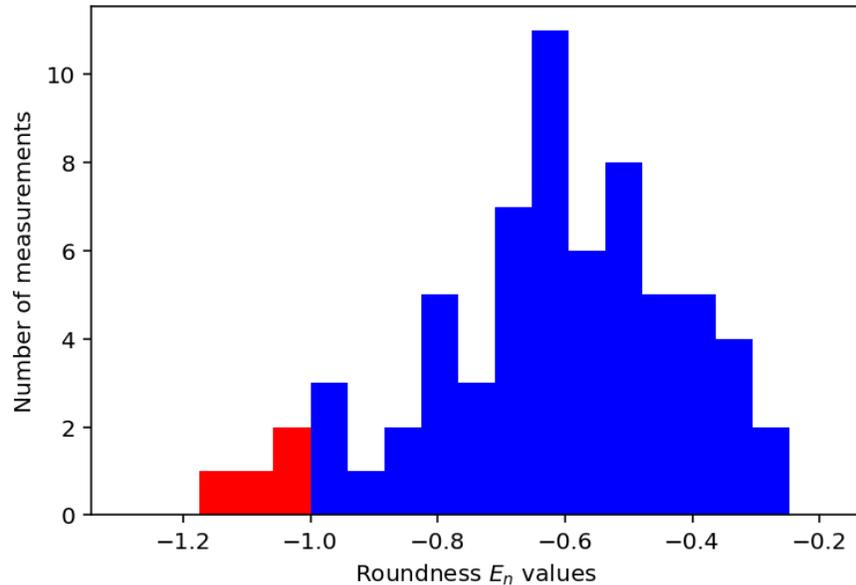| Method	Coverage-% Radius	Coverage-% Roundness
Standard procedure	93 %	18 %
A. Bias correction of estimate	93 %	95 %
B. Increasing uncertainty with bias	94 %	95 %

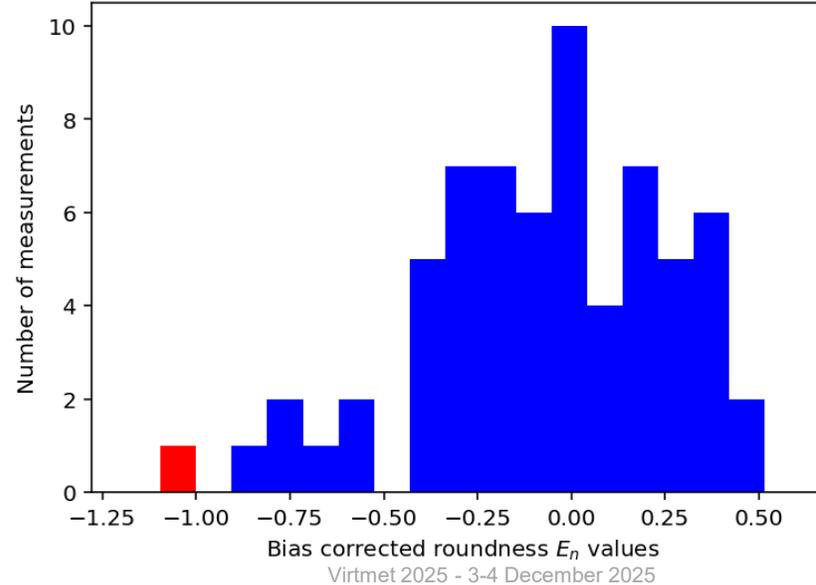# Experimental validation of both methods for dealing with bias

$$E_n = \frac{x_{\text{ref}} - x_i}{k\sqrt{u^2(x_{\text{ref}}) + u^2(x_i)}}$$
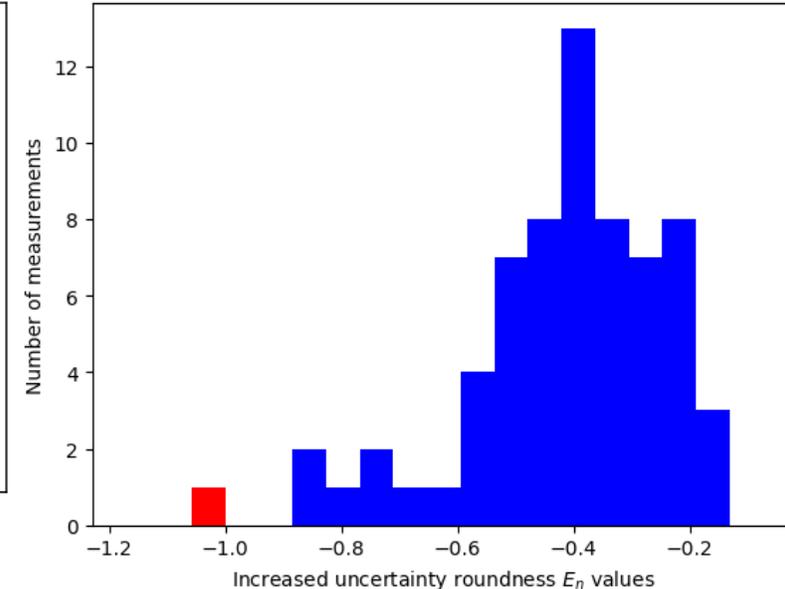
Original results:
- negatively biased
- several inconsistencies

A. After correcting the estimate for the bias: Only 1 inconsistency, more symmetric distribution

B. After increasing the uncertainty: Only 1 inconsistent result left

Conclusion

1. In the case of smooth measurands, all uncertainty evaluations methods give very similar results.

2. Be careful with what is meant by the term "VCMM":
    a) Is it a simulation of measurement data: $X = g_1(Y, Z)$?
    b) Is it perturbing measurement data with additional measurement errors: $X_\text{out} = g_2(X_\text{in}, Z)$?
    c) Is it a complete data evaluation chain: $u(\hat{Y}_\text{out}) = g_3(X_\text{in}, Z)$ or $(\hat{Y}_\text{out}, u(\hat{Y}_\text{out})) = g_4(X_\text{in}, Z)$?

3. Statistical biases in data analysis approaches matter and should be taken into account.
    a. Either correct your estimate (be aware of conservative uncertainties though)
    b. Or increase your uncertainty

4. More lessons learnt from CMM case study to appear in ViDiT WP1 GPG ! (www.vidit.eu)

Any questions?

Recent publications

- Gertjan Kok, Gerd Wübbeler, Clemens Elster, *Impact of imperfect artefacts and the modus operandi on uncertainty quantification using virtual instruments*, Metrology, 2022, 2(2), 311-319; https://doi.org/10.3390/metrology2020019

- Gertjan Kok, Marcel van Dijk, Gerd Wübbeler and Clemens Elster, *Virtual experiments for the assessment of data analysis and uncertainty quantification methods in scatterometry*, Metrologia 60 044001, DOI 10.1088/1681-7575/acd6fd, https://iopscience.iop.org/article/10.1088/1681-7575/acd6fd

- Marcel van Dijk, Gertjan Kok, *Comparison of uncertainty evaluation methods for virtual experiments with an application to a virtual CMM*, Measurement: Sensors, 2025, 101785, ISSN 2665-9174, https://doi.org/10.1016/j.measen.2024.101785

- Gertjan Kok and Marcel van Dijk, *Measurement Uncertainty Evaluation: Differences between Virtual Experiments and the Standardized Approach*, Metrology. 2025; 5(4):59. https://doi.org/10.3390/metrology5040059

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